1.2 EXERCISES

In Exercises 1 and 2, determine which matrices are in reduced echelon form and which others are only in echelon form.

1. a. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$ b. $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ c. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ d. $\begin{bmatrix} 1 & 1 & 0 & 1 & 1 \\ 0 & 2 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 0 & 4 \end{bmatrix}$

2. a. $\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ b. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$ c. $\begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ d. $\begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

Row reduce the matrices in Exercises 3 and 4 to reduced echelon form. Circle the pivot positions in the final matrix and in the original matrix, and list the pivot columns.

3. $\begin{bmatrix} 1 & 2 & 4 & 8 \\ 2 & 4 & 6 & 8 \\ 3 & 6 & 9 & 12 \end{bmatrix}$ **4.** $\begin{bmatrix} 1 & 2 & 4 & 5 \\ 2 & 4 & 5 & 4 \\ 4 & 5 & 4 & 2 \end{bmatrix}$

5. Describe the possible echelon forms of a nonzero 2×2 matrix. Use the symbols ■, $*$, and 0, as in the first part of Example 1.

6. Repeat Exercise 5 for a nonzero 3×2 matrix.

Find the general solutions of the systems whose augmented matrices are given in Exercises 7–14.

7. $\begin{bmatrix} \mathbf{1} & 3 \\ \mathbf{3} & 9 \end{bmatrix}$ **8.** $\begin{bmatrix} 1 & -3 & 0 & -5 \\ \mathbf{-3} & 7 & & \end{bmatrix}$

9. $\begin{bmatrix} 0 & 1 & -2 & 3 \\ 1 & -3 & 4 & -6 \end{bmatrix}$ **10.** $\begin{bmatrix} 1 & -2 & -1 & 4 \\ -2 & 4 & -5 & 6 \end{bmatrix}$

11. $\begin{bmatrix} 3 & -2 & 4 & \\ 9 & -6 & 12 & 0 \\ 6 & -4 & 8 & \end{bmatrix}$ **12.** $\begin{bmatrix} 1 & 0 & -9 & 0 & \\ \mathbf{0} & 1 & & & -1 \\ \mathbf{0} & 0 & & & -7 \\ \mathbf{0} & 0 & & & 1 \end{bmatrix}$

16. a. $\begin{bmatrix} ■ & * & * \\ 0 & ■ & * \\ 0 & 0 & ■ \end{bmatrix}$ b. $\begin{bmatrix} ■ & * & * & * & * \\ 0 & 0 & ■ & * & * \\ 0 & 0 & 0 & ■ & * \end{bmatrix}$

In Exercises 17 and 18, determine the value(s) of h such that the matrix is the augmented matrix of a consistent linear system.

17. $\begin{bmatrix} 1 & -1 & 4 \\ -2 & 3 & h \end{bmatrix}$ **18.** $\begin{bmatrix} 1 & -3 & 1 \\ h & 6 & -2 \end{bmatrix}$

In Exercises 19 and 20, choose h and k such that the system has (a) no solution, (b) a unique solution, and (c) many solutions. Give separate answers for each part.

19. $x_1 + hx_2 = 2$
$4x_1 + 8x_2 = k$

20. $x_1 - 3x_2 = 1$
$2x_1 + hx_2 = k$

In Exercises 21 and 22, mark each statement True or False. Justify each answer.[4]

21. a. In some cases, a matrix may be row reduced to more than one matrix in reduced echelon form, using different sequences of row operations.

b. The row reduction algorithm applies only to augmented matrices for a linear system.

c. A basic variable in a linear system is a variable that corresponds to a pivot column in the coefficient matrix.

d. Finding a parametric description of the solution set of a linear system is the same as *solving* the system.

e. If one row in an echelon form of an augmented matrix is $[\,0 \quad 0 \quad 0 \quad 5 \quad 0\,]$, then the associated linear system is inconsistent.

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13. $\begin{bmatrix} 1 & -3 & 0 & -1 & 0 & -2 \\ 0 & 1 & 0 & 0 & -4 & 1 \\ 0 & 0 & 0 & 1 & 9 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

14. $\begin{bmatrix} 1 & 0 & -5 & 0 & -8 & 3 \\ 0 & 1 & 4 & -1 & 0 & 6 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

Exercises 15 and 16 use the notation of Example 1 for matrices in echelon form. Suppose each matrix represents the augmented matrix for a system of linear equations. In each case, determine if the system is consistent. If the system is consistent, determine if the solution is unique.

15. a. $\begin{bmatrix} \blacksquare & * & * & * \\ 0 & \blacksquare & * & * \\ 0 & 0 & 0 & 0 \end{bmatrix}$

b. $\begin{bmatrix} 0 & \blacksquare & * & * & * \\ 0 & 0 & \blacksquare & * & * \\ 0 & 0 & 0 & \blacksquare & 0 \end{bmatrix}$

22. a. The reduced echelon form of a matrix is unique.

b. If every column of an augmented matrix contains a pivot, then the corresponding system is consistent.

c. The pivot positions in a matrix depend on whether row interchanges are used in the row reduction process.

d. A general solution of a system is an explicit description of all solutions of the system.

e. Whenever a system has free variables, the solution set contains many solutions.

23. Suppose the coefficient matrix of a linear system of four equations in four variables has a pivot in each column. Explain why the system has a unique solution.

24. Suppose a system of linear equations has a 3×5 *augmented* matrix whose fifth column is not a pivot column. Is the system consistent? Why (or why not)?

[4] True/false questions of this type will appear in many sections. Methods for justifying your answers were described before Exercises 23 and 24 in Section 1.1.

25. Suppose the coefficient matrix of a system of linear equations has a pivot position in every row. Explain why the system is consistent.

26. Suppose a 3×5 *coefficient* matrix for a system has three pivot columns. Is the system consistent? Why or why not?

27. Restate the last sentence in Theorem 2 using the concept of pivot columns: "If a linear system is consistent, then the solution is unique if and only if _____."

28. What would you have to know about the pivot columns in an augmented matrix in order to know that the linear system is consistent and has a unique solution?

29. A system of linear equations with fewer equations than unknowns is sometimes called an *underdetermined system*. Can such a system have a unique solution? Explain.

30. Give an example of an inconsistent underdetermined system of two equations in three unknowns.

31. A system of linear equations with more equations than unknowns is sometimes called an *overdetermined system*. Can such a system be consistent? Illustrate your answer with a specific system of three equations in two unknowns.

32. Suppose an $n \times (n+1)$ matrix is row reduced to reduced echelon form. Approximately what fraction of the total number of operations (flops) is involved in the backward phase of the reduction when $n = 20$? when $n = 200$?

Suppose experimental data are represented by a set of points in the plane. An **interpolating polynomial** for the data is a polynomial whose graph passes through every point. In scientific work, such a polynomial can be used, for example, to estimate values between the known data points. Another use is to create curves for graphical images on a computer screen. One method for finding an interpolating polynomial is to solve a system of linear equations.

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33. Find the interpolating polynomial $p(t) = a_0 + a_1 t + a_2 t^2$ for the data $(1, 6)$, $(2, 15)$, $(3, 28)$. That is, find a_0, a_1, and a_2 such that

$$a_0 + a_1(1) + a_2(1)^2 = 6$$
$$a_0 + a_1(2) + a_2(2)^2 = 15$$
$$a_0 + a_1(3) + a_2(3)^2 = 28$$

34. [**M**] In a wind tunnel experiment, the force on a projectile due to air resistance was measured at different velocities:

Velocity (100 ft/sec)	0	2	4	6	8	10
Force (100 lb)	0	2.90	14.8	39.6	74.3	119

Find an interpolating polynomial for these data and estimate the force on the projectile when the projectile is traveling at 750 ft/sec. Use $p(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5$. What happens if you try to use a polynomial of degree less than 5? (Try a cubic polynomial, for instance.)[5]

[5] Exercises marked with the symbol [**M**] are designed to be worked with the aid of a "**M**atrix program" (a computer program, such as **M**ATLAB®, **M**aple™, **M**athematica®, **M**athCad®, or Derive™, or a programmable calculator with matrix capabilities, such as those manufactured by Texas Instruments or Hewlett-Packard).

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